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AN ADI DIFFERENCE SCHEME FOR 3D PROBLEMS OF TENSOR HEAT CONDUCTION

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A difference scheme for calculating temperature distribution is elaborated to fit requirements to be monotone, conservative and exponential with ADI algorithm for 3D grid. The main benefit of this difference scheme is reduced calculation time based on algorithm blocks which are not connected between themselves. It is quite advantage for 3D calculations.

The main problem of implementing the difference scheme was connected to the heat conduction tensor.

Termodispersion tensor is rewritten in exponential way with connections between the axis of the system. This approach makes very sensitive system to the input data.

This 3D temperature distribution difference equation is used:

$$
\begin{aligned}
&\frac{\partial T}{\partial t} + V_x\frac{\partial T}{\partial x} + V_y\frac{\partial T}{\partial y} + V_z\frac{\partial T}{\partial z}\\
&\frac{1}{R_t}\frac{\partial}{\partial x}\left(\Lambda_{xx}\frac{\partial T}{\partial x} + \Lambda_{xy}\frac{\partial T}{\partial y} + \Lambda_{xz}\frac{\partial T}{\partial z}\right) + \frac{1}{R_t}\frac{\partial}{\partial y}\left(\Lambda_{yx}\frac{\partial T}{\partial x} + \Lambda_{yy}\frac{\partial T}{\partial y} + \Lambda_{yz}\frac{\partial T}{\partial z}\right) +\\
&+\frac{1}{R_t}\frac{\partial}{\partial z}\left(\Lambda_{zx}\frac{\partial T}{\partial x} + \Lambda_{zy}\frac{\partial T}{\partial y} + \Lambda_{zz}\frac{\partial T}{\partial z}\right)
\end{aligned}
$$

with comforting beginning and boundary conditions.

By discretisation of the difference equation we get the differential scheme for temperature calculations from the time moment "l" to the time moment "l+1":

$$
\begin{aligned}
&\frac{\overset{l+1}{T}_{ijk} - \overset{l}{T}_{ijk}}{\tau} = \frac{1}{R_t}\frac{(I_x)_{i+\frac{1}{2}jz} - (I_x)_{i-\frac{1}{2}jz}}{h*_x} +\\
&+\frac{1}{R_t}\frac{(I_y)_{ij+\frac{1}{2}z} - (I_y)_{ij-\frac{1}{2}z}}{h*_y} + \frac{1}{R_t}\frac{(I_z)_{ijz+\frac{1}{2}} - (I_z)_{ijz-\frac{1}{2}}}{h*_z} + \frac{QT_{ijk}}{nR_t}
\end{aligned}
$$

where averaged flow in x direction can be found in exponential way:

$$
\begin{aligned}
&(I_x)_{i+\frac{1}{2}jk} = \Lambda_{xx}\frac{Exp\left[p_{i+\frac{1}{2}jk}\right]T_{i+1jk} - T_{ijk}}{p_{i+\frac{1}{2}jk}\left(Exp\left[p_{i+\frac{1}{2}jk}\right]-1\right)};\\
&p_{i+\frac{1}{2}jk} = \frac{1}{T_{i+1jk}\Lambda_{xx}}\left(\Lambda_{xy}\frac{T_{ij+1k} - T_{ijk}}{h_y} + \Lambda_{xz}\frac{T_{ijk+1} - T_{ijk}}{h_z} - v_x T_{i+\frac{1}{2}jk}\right);\\
&(I_x)_{i-\frac{1}{2}jk} = \Lambda_{xx}\frac{Exp\left[p_{i-\frac{1}{2}jk}\right]T_{ijk} - T_{i-1jk}}{p_{i-\frac{1}{2}jk}\left(Exp\left[p_{i-\frac{1}{2}jk}\right]-1\right)};\\
&p_{i-\frac{1}{2}jk} = \frac{1}{T_{i-1jk}\Lambda_{xx}}\left(\Lambda_{xy}\frac{T_{ijk} - T_{ij-1k}}{h_y} + \Lambda_{xz}\frac{T_{ijk} - T_{ijk-1}}{h_z} - v_x T_{i-\frac{1}{2}jk}\right);
\end{aligned}
$$

In the same way the averaged flow can be written for y and z directions. For these calculations the ADI method is used. ADI method guarantee that every dimension every equation component results can be calculated totally separately from every dimension every equation component aspect.

The advised mathematical model, its effective calculation algorithm and developed application code for parallel calculation technologies can be applied to non-stationary heat process of environmental impact assessment anizantrope 3D space. The method which is based on the matrix method needs too much amount of memory and takes too much machine time for making the calculations, so it is assumed to be totally impracticable.